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Life Expectancies of Couples by Age Combination of Spouses

1. The Problem

IN an earlier paper, Mitra (1977) has shown that the eventual probability of becoming a widow or a widower depends primarily on the age difference of the two spouses and not so much on their actual ages. This was shown by first noting that the probability $P(a, b)$ that a husband aged a will outlive his wife aged b can be expressed as

$$P(a,b) = \frac{\int_0^{\alpha} l_m(a+x)\, l_f(b+x)\, \mu_f(b+x)\, dx}{l_m(a)\, l_f(b)} \qquad (1)$$

in which $l_m(a+x)$ is the proportion of male survivors from birth to age $a+x$, $l_f(b+x)$ is the corresponding female function, μ is the force of mortality and α is the upper limit of x, at which one of the two l functions is the first to become zero. Assuming Gompertz law of mortality, that is

$$\mu_m(a) = -\frac{1}{l_m(a)} \frac{dl_m(a)}{da} = B_m C^a \qquad (2)$$

$$\mu_f(b) = -\frac{1}{l_f(b)} \frac{dl_f(b)}{db} = B_f C^b \qquad (3)$$

where the equality of the parameter C for the two sexes was noted as an empirical fact, it was proved that (1) reduces to

$$P(a, b) = \frac{1}{1 + (B_m/B_f)\, C^{a-b}} \tag{4}$$

The complementary probability measuring the proportion of cases in which the husband will be outlived by their wives is then given by

$$Q(a, b) = 1 - P(a, b) \tag{5}$$

It follows that the probabilities generated by (4) depend only on the age difference between the two spouses and not on their actual ages. However, as one would expect, *P (a, b)* decreases as the age difference increases, since C>1 is another empirical fact. Using U. S. examples, the probabilities generated by (4) were found to differ little from the actual values derived directly from (1) by numerical integration, for virtually all possible age combinations. For the convenience of the readers, these results presented earlier (Mitra, 1977) are reproduced in Table 1.

TABLE 1— PROBABILITY *P (a, b)* OF HUSBAND *a* YEARS OLD, OUTLIVING THE WIFE AGED *b* YEARS, ESTIMATED BY THE METHODS BASED ON (1) GOMPERTZ LAW OF MORTALITY AND (2) NUMERICAL INTEGRATION (SOURCE : 1973 U. S. LIFE TABLES)

P (a, b) by Gompertz law			*Optimum values of P (a, b) by Numerical Integration*			
a — b	*a > b*	*a < b*	*a ⩾ b* Minimum	*a ⩾ b* Maximum	*a ⩽ b* Minimum	*a ⩽ b* Maximum
(0	(2)	(3)	(4)	(5)	(6)	(7)
0	.370	.370	.345	.382	.345	.382
1	.350	.390	.327	.361	.364	.398
2	.331	.410	.308	.341	.383	.415
3	.312	.431	.291	,321	.403	.433
4	.295	.452	.273	.301	.422	.452
5	.277	.473	.256	.282	.443	.472
6	.261	.494	.240	.263	.463	.492
7	.244	.515	.224	.245	.484	.512
8	.229	.536	.209	.227	.504	.532
9	.215	.557	.196	.211	.525	.551
10	.201	.578	.183	.195	.545	.571

The purpose of this paper is to carry on the investigation a step farther by evaluating the number of years $A(a, b)$ that a couple can expect to live together until death breaks it apart, and the expected number of years $H(a, b)$ [$W(a, b)$] that the husband (wife) will survive as a widower (widow).

2. Functional Relationships

What follows next is an explicit formulation of the relationships that the parameters defined so far, bear with the standard life table functions and also with themselves. In particular,

$$A(a, b) = \frac{\int_0^{\alpha} l_m(a + x)\, l_f(b + x)\, dx}{l_m(a)\, l_f(b)} \tag{6}$$

as it provides a measure of the number of years that a husband and wife aged a and b years, respectively can expect to survive as a couple. In order to develop an expression for $H(a, b)$, one has first to consider the probability of the husband surviving x years after age a and the wife dying at age $b + x$, which is

$$\frac{l_m(a + x)}{l_m(a)} \; \frac{l_f(b + x)}{l_f(b)} \; \mu_f(b + x) \tag{7}$$

and then to note that the husband can thereafter expect to live $e_m(a + x)$ more years (the male life expectancy at age $a + x$), as a widower. Thus,

$$H(a, b) = \frac{\int_0^{\alpha} l_m(a + x)\, l_f(b + x)\, \mu_f(b + x)\, e_m(a + x)\, dx}{l_m(a)\, l_f(b)\, P(a, b)}$$

$$= \frac{-\int_0^{\alpha} T_m(a + x)\, l_f'(b + x)\, dx}{l_m(a)\, l_f(b)\, P(a, b)} \tag{8}$$

where $T_m(a + x) = \int_0^{\infty} l_m(a + x)\, dx = l_m(a + x)\, e_m(a + x)$ (9)

and $l_f'(b + x) = dl_f(b + x)/dx = -\, l_f(b + x)\, \mu_f(b + x)$ (10)

and the upper limit of the integral of $T_m(a + x)$ is written as infinity for operational convenience. The appearance of $P(a,b)$ in the denominator of (8) is due to the fact that the average is applied only to those husbands who had outlived their wives, the proportion of such cases being $P(a, b)$. In other words, $H(a, b)$ is conditional to widowhood as it provides the average number of years a person can expect to live after losing his wife.

The integral appearing in the numerator of (8) can be alternatively expressed as

$$T_m(a)\, l_f(b) - \int_0^{\alpha} l_m(a+x)\, l_f(b+x)\, dx \tag{11}$$

after integration by parts. Substitution of (11) in (8) results in

$$A(a, b) + P(a, b)\, H(a, b) = e_m(a) \tag{12}$$

after rearrangement of terms and because of (6). Starting with the wives, the corresponding expression

$$A(a, b) + Q(a, b)\, W(a, b) = e_f(b) \tag{13}$$

is obtained.

It is interesting to note that the relationship expressed in (12) and (13) can also be arrived at by simple probabilistic logic that is characteristic of the situation. The argument is developed by noting that $P(a,b)$ of the husbands who become widowers after spending $A(a, b)$ years with their wives can expect to live an additional $H(a, b)$ years on an average. Similarly, $Q(a, b)$ stands for the proportion of the cases in which the husbands, who died first, lived an average of $A(a,b)$ years. Combining these two components and noting that their sum must be equal to the male life expectancy at age a, it is possible to write

$$P(a, b)\,[A(a, b) + H(a, b)] + Q(a, b)\,A(a, b) = e_m(a) \tag{14}$$

which after simplification reduces to (12).

3. Application of Results

Given $P(a, b)$ and the solution of $A(a, b)$ from (6), $H(a, b)$ and $W(a, b)$ can be solved from (12) and (13). That is to say, these parameters need not be directly obtained from expressions like (8) which are computationally inconvenient and unnecessary because of the foregoing results. What follows next is a demonstration of the method on the 1973 U. S. life tables.

The first major task is the estimation of the numerator of A (a, b), as shown in (6). The task is simplified by first assuming that for life table functions available by single year of age,

$$\int_{t}^{t+1} l_m(a+x)\, l_f(b \quad x)\, dx \simeq \tfrac{1}{2}\,[l_m(a+t+1)\, l_f(b+t+1)$$
$$+\, l_m(a+t)\, l_f(b+0] \quad 015)$$

where $\simeq$ stands for an approximate equality. Next, adjustments for the last open-ended interval (85 +) for the / functions were made by projecting their values up to age 100, assuming the continuation of Gompertz law. Thereafter the computations of the values of A (a, b) and then those of H (a, b) and $W(a,b)$ for different age combinations of the spouses were easily carried out by. the electronic computer. These values, for selected age combinations of the spouses are shown in Table 2 using the estimates of $P(a, b)$ and $Q(a, b)$ generated by the Gompertz law and shown in Table 1.

As could be expected, for a fixed value of husband's age a, $A(a, b)$ decreases with increase in wife's age b and vice versa. That is to say, the number of years a couple can expect to live together is inversely related to the age of one or both spouses. The pattern is the same for $W(a, b)$ but is reversed for $H(a, b)$. This is also expected since $W(a, b)$, the number of years a woman can expect to live as a widow, must decrease as tbe difference $b - a$ gets larger and larger. The opposite trend in $H(a, b)$ can be similarly explained.

4. Estimate by Gompertz Law

It may be pointed out that the derivation of the values of $A(a, b)$ were made in a straightforward manner except for the fact that the Gompertz law of mortality was explicitly assumed to operate beyond the age of 85 for which the values of the life table functions were not available. The effects of these adjustments on the estimates of $A(a, b)$ are expected to be minimal and for all practical purposes, these estimates can be treated as reasonably close approximation* of the actual values. However, the same may not be said about the estimates Of the other two functions, namely $H(a, b)$ and $W(a, b)$ which are shown in Table 2. This is so, because the values of $P(a, b)$ and $Q(a, b)$ used in their evaluations were derived from the assumption of the Gompertz model. The question that naturally arises in this context, revolves around the possibility Of reducing the algebraic expression of A (a, 6) by the same law and this ha* been dealt with in the following.

TABLE 2-VALUES OF EXPECTANCIES *A*, **(*a*, *b*)**, ***H***(*a*, *b*) AND ***W***(*a*, *b*) BY **HUSBAND'S AGE *a* AND HUSBAND-WIFE** AGE DIFFERENCE *a* – ***b*** **: U. S.** (1973)

Husband's Age	***Expectancies***	*Values by husband-wife age difference (years)*							
		-5	*-3*	*-1*	*0*	*1*	*3*	5	*S*
	A	43.1	43.9	44.6	44.9	45.2	x	X	x
20	H	**14.6**	14.1	31.7	13.5	13.3	x	X	x
	W	17.3	18.0	18.8	**19.3**	19.7	x	x	x
	A	38.7	39.5	40.2	40.6	40.9	41.4	41.9	x
25	H	14.3	13.8	**13 3**	13.1	12.9	12.6	12.4	x
	W	16.5	17.3	18.1	18.6	19.0	20.0	21.1	x
	A	**34.3**	35.1	35.8	36.1	36.4	37.0	37.5	38.1
30	H	**4.11**	13.6	13.2	13.0	12.8	12.5	12.2	11.9
	W	**15.8**	16.7	17.6	18.1	18.5	19.4	20.6	22.2
	A	29.9	30.7	31.4	31.7	31.9	32.5	33.0	33.6
35	H	13.7	13.2	12.7	12.5	12.4	12.0	**11.7**	11.4
	W	15.2	16.2	17.1	17.5	17.9	19.0	20.2	**21.8**
	A	25.7	26.4	27.1	27.4	27.7	28.2	28.7	29.3
40	H	**13.1**	12.6	12.2	12.0	18.8	11.5	**11.2**	10.9
	W	**14.7**	15.6	16.4	16.9	17.5	18.4	19.5	21.2
	A	12.7	22.4	23.0	**23.3**	**23.5**	24.0	24.5	25.0
45	H	12.4	12.0	**11.6**	11.5	11.3	**11.0**	10.8	**10.5**
	W	13.9	14.9	15.7	16.2	16.8	17.8	18.8	20.6
	A	18.0	***18.6***	19.2	19.5	19.7	20.2	20.6	21.1
50	H	11.5	11.2	10.8	10.6	10.5	10.2	10.0	9.8
	W	13.0	13.9	14.9	15.3	15.8	16.8	18.0	19.7
	A	**14.6**	15.2	15.7	16.0	16.2	16.6	17.0	17.5
55	H	**10.7**	10.3	10.0	9.9	9.8	9.5	9.3	**9.1**
	W	11.8	12.8	13.7	14.2	14.7	15.8	16.8	18.6
	A	11.6	12.2	12.6	12.9	13.1	13.5	13.8	14.2
60	H	9.8	9.5	9.2	9.0	8.9	8.7	8.6	8.4
	W	10.5	11.3	12.4	12.8	13.3	14.4	15.4	17.2
	A	9.0	9.5	9.9	10.1	103	10.7	11.0	11.4
65	H	8.8	8.5	8.2	8.0	7.9	7.7	7.5	7.3
	W	8,7	9.7	10.6	11.2	11.7	12.7	13.8	15.5
	A	6.7	7.1	7.5	7.7	7.9	8.2	8.5	8.9
70	H	7.9	7.6	7.4	7.3	7.1	**6.9**	6.6	6.4
	W	7.1	8.0	8.8	9.3	9.9	10.9	12.0	***13.7***
	A	5.0	5.3	5.6	5.7	**5.9**	6,2	6.4	6.8
75	H	6.9	6.8	6.7	6.7	6.6	6.5	6.3	6,1
	W	5.5	6.3	7.1	7.6	8.0	9.0	10.0	11.7

First, the well known relationships between the l and μ functions given by

$$l_m (a + x) = e^{-\int \mu_m (a + x)\, dx} \tag{16}$$

for the males and

$$l_f (b + x) = e^{-\int \mu_f (b + x)\, dx} \tag{17}$$

for the females are simplified by the application of Gompertz law (2 and 3) as

$$l_m (a + x) = e^{-B_m C^a \int C^x dx} \tag{18}$$

and

$$l_f (b + x) = e^{-B_f C^b \int C^x dx} \tag{19}$$

A comparison of (18) and (19) may then be expressed as

$$[l_m (a + x)]^{B_f C^b} = [l_f (b + x)]^{(B_m C^a)} \tag{20}$$

or as

$$l_m (a + x) = [l_f (b + x)]^{(B_m/B_f) C^{a-b}} \tag{21}$$

Noting from (4) and (5) that

$$Q (a, b)/P (a, b) = (B_m/B_f) C^{a-b} \tag{22}$$

(21) can be written as

$$[(l_m (a + x)]^{P (a, b)/Q (a, b)} = l_f (b + x) \tag{23}$$

or as

$$[l_m (a + x) = [l_f (b + x)]^{Q (a, b)/P (a, b)} \tag{24}$$

Substitution of (23) and (24) in (5) results in

$$A (a, b) = \frac{\int_0^{\alpha} [l_m (a + x)]^{1/Q (a, b)} dx}{[l_m (a)]^{1/Q (a, b)}} = \frac{\int_0^{\alpha} [l_f (b + x)]^{1/P (a, b)} dx}{[l_f (b)]^{1/P (a, b)}} \tag{25}$$

The simplification of (5) provides the answer to the question raised at the beginning of the section. It may be reiterated that the upper limit of x, namely α, in either of the integrals of- (25) is of course determined by that age at which $l_m(a+x)$ or $l_f(b+x)$ is the first to become zero.

The interesting feature of (25) is that the value of $A(a, b)$ can be obtained from either the male or the female life tables whereas both life tables arc required (see 6) for its evaluation in a straightforward manner. A closer look at (25) reveals that $A(a, b)$ is like the life expectancy function of a hypothetical life table for which the survivorship function is

$$[l_m(a+x)]^{1/Q(a,b)} \quad \text{or} \quad [l_f(b+x)]^{1/P(a,b)}$$

The formula is interesting from a mathematical point of view, although, it may not offer any computational advantage over its alternative, described earlier. It may also be noted that (25) evaluated from the male life table, can be expected to differ slightly from the same obtained from the female life table because the Gompertz formula is not exact and is, at best, an approximation for the force of mortality. The difference for a few selected age combinations was found to be less than one year and that somewhat obliquely, attests to the fairness of Gompertz approximation, at least with the U. S. example chosen for this study.

5. Further Analysis

As noted earlier (see Table 2), the function $A(a, b)$ is inversely related with the age of either spouse. However, because of mortality differentials by sex, the changes in $A(a, b)$ for similar changes in husband's and in wife's age will usually be different. A systematic investigation of this relationship is possible through an analysis of the derivatives of the functions that determine the fundamental equations (12) and (13), and this has been attempted next.

First the partial derivative of $P(a, b)$ can be obtained from (1) as

$$\frac{\partial P(a,b)}{\partial a} = X + P(a,b)\,\mu_m(a) \tag{26}$$

where

$$X = \frac{-\int_0^{\alpha} l_m(a+x)\, l_f(b+x)\, \mu_m(a+x)\, \mu_f(b+x)\, dx}{l_m(a)\, l_f(b)} \tag{27}$$

"and the assumption of continuity etc. holds so that the differentiation can be carried on within the integral. Interchanging P with Q and a with b, the par-

tial derivative of Q with respect to b can be expressed as

$$\frac{\partial Q(a, b)}{\partial b} = \ldots\, Q(a, b)\, \mu_f(b). \tag{28}$$

The other two derivatives can be obtained from **(5) as**

$$\frac{\partial P(a, b)}{\partial a} = - \frac{\partial Q(a, b)}{\partial a} \tag{29}$$

$$\frac{\partial P(a, b)}{\partial b} = - \frac{\partial Q(a, b)}{\partial b}. \tag{30}$$

Further, when the Gompertz law of mortality is assumed, **all** four partial **derivatives** become equal to one another in terms of absolute values. In **fact,**

$$\frac{\partial P(a, b)}{\partial a} = \frac{\partial Q(a, b)}{\partial b} = - \frac{\partial P(a, b)}{\partial b} = - \frac{\partial Q(a, b)}{\partial a}$$
$$= - \ln c\, P(a, b)\, Q(a, b) \tag{31}$$

can be obtained directly from (4). The equality of all four of **these** partial derivatives follows from the fact that (4) is **also** a function of $a-b$, the age **difference** of the two **spouses.**

The derivatives of $H(a, b)$ can be easily examined through those of $P(a, b)\, H(a, b)$ which can be obtained from (8) as

$$P(a, b)\, H(a, b) = \frac{- \int_0^\infty T_m(a + x)\, l_f'(b + x)\, dx}{l_m(a)\, l_f(b)}. \tag{32}$$

The partial derivative with respect to a of the **l.h.s.** of (32) is

$$P(a, b) \frac{\partial H(a, b)}{\partial a} + H(a, b) \frac{\partial P(a, b)}{\partial a} \tag{33}$$

and that of the **r.h.s.** is

$$\frac{\int_0^\infty l_m(a + x)\, l_f'(b + x)\, dx}{l_m(a)\, l_f(b)} + P(a, b)\, H(a, b) \left[- \frac{1}{l_m(a)} \frac{dl_m(a)}{da} \right] \tag{34}$$

which simplifies to

$$P(a, b)\,[H(a, b)\,\mu_m(a) - 1] \tag{35}$$

because $\frac{d}{da} T_m(a+x) = -l_m(a+x)$,

and further the first term of (34) is $-P(a, b)$ as may be seen from (1). Equating (33) with (35) and simplifying,

$$\frac{\partial H(a, b)}{\partial a} = H(a, b)\,\mu_m(a) - 1 - \frac{H(a, b)}{P(a, b)} \frac{\partial P(a, b)}{\partial a} \tag{36}$$

which because of (26) reduces further to

$$\frac{\partial H(a, b)}{\partial a} = -\frac{XH(a, b)}{P(a, b)} - 1. \tag{37}$$

Using Gompertz law, $\partial P(a, b)/\partial a$ in (36) can be replaced by (31), in which case

$$\frac{\partial H(a, b)}{\partial a} = H(a, b)\,[\mu_m(a) + Q(a, b) \ln C] - 1. \tag{38}$$

The values of the other partial derivative may be similarly obtained.

The stage is now set for the evaluation of the partial derivatives of A. First, it is possible to write from (12)

$$\frac{\partial A(a, b)}{\partial a} + \frac{\partial [P(a, b)\, H(a, b)]}{\partial a} = \frac{d}{da} e_m(a). \tag{39}$$

The r.h.s. of (39) can be written as (Mitra, 1974)

$$\frac{d}{da}\left[\frac{T_m(a)}{l_m(a)}\right] = \frac{-l_m(a)}{l_m(a)} + \frac{T_m(a)}{l_m(a)}\left[-\frac{1}{l_m(a)} \frac{dl_m(a)}{da}\right]$$
$$= e_m(a)\,\mu_m(a) - 1 \tag{40}$$

so that (39) can be written as

$$\frac{\partial A(a, b)}{\partial a} = e_m(a)\,\mu_m(a) - 1 - P(a, b)\,[H(a, b)\,\mu_m(a) - 1] \tag{41}$$

because of (35). Simplifying further,

$$\frac{\partial A(a, b)}{\partial a} = A(a, b)\,\mu_m(a) - Q(a, b) \tag{42}$$

is obtained because of (12). A similar treatment of (13) results in

$$\frac{\partial A(a, b)}{\partial b} = A(a, b)\,\mu_f(b) - P(a, b), \tag{43}$$

According to Gompertz law (sec 2-5),

$$\frac{\mu_m(a)}{\mu_f(b)} = \frac{Q(a, b)}{P(a, b)} \tag{44}$$

in which case, (43) dan also be expressed as

$$\begin{aligned}
\frac{\partial A(a, b)}{\partial b} &= \frac{A(a, b)\,P(a, b)\,\mu_m(a)}{Q(a, b)} - P(a, b) \\
&= \frac{P(a, b)}{Q(a, b)}\left[A(a, b)\,\mu_m(a) - Q(a, b)\right] \\
&= \frac{P(a, b)}{Q(a, b)}\,\frac{\partial A(a, b)}{\partial a}
\end{aligned} \tag{45}$$

because of (42). This relationship is useful in comparing the rates of change of A effected by a change in husband's age alone with that caused by a change only in wife's age.

6: Comparison with Actual Life Experiences

Looking back to the definition of $A(a, b)$ in (6), it becomes clear that $A(a, b)$ can be evaluated for the specific age combination of a couple and in that sense it as specific to that particular marriage regardless of the date of marriage. In other words, $A(a, b)$ would measure the expected additional number of years that a husband presently aged a will live as married to the wife whose present age is b and vice versa provided that a marriage is terminated only by the death of. one of the spouses and not otherwise. in real life, however, a marriage is dissolved not only through death, but also through divorce and a significant proportion of widows, widowers and divorcees also remarry. Accordingly, it will be quite interesting to see how the experiences in real life compare with that generated by the simple model characterized by the absence of divorce and serial marriages. For such comparisons, it is necessary to decompose the life expectancies into the different marital statuses. Such decompositions can be obtained (Mitra, *et al.*, 1977) by assuming that the life expectancy of the married population is not different from that of the general population and further that the distribution by age, sex and marital status in a population is similar to that in the corresponding stationary population. Based on such simplifying assumptions, the values of $A(a, b)$ have been compared with $B(a, 6)$, the expected number of years to be lived as married (allowing for divorce and remarriage), for the U.S. population in Table 3, for selected values of husband's age and the normative husband-wife age differences of 2 and 3 years.

While analyzing the figures in Table 3, it should be remembered that the upper limit of the number of years that a man, *a* years old, can spend as married in

TABLE 3—NUMBER OF YEARS ONE CAN EXPECT TO LIVE AS MARRIED ALLOWING FOR DIVORCE AND REMARRIAGE i.e., $B(a, b)$ COMPARED WITH THE SAME WHEN DIVORCE AND REMARRIAGE ARE NOT PERMITTED i.e., $A(a, b)$: BASED ON U. S. 1970 CENSUS AGE-SEX COMPOSITION

Husband's age a	$e_m(a)$	*A (a, b) when a — b is*		*B (a, 6)*		
		2 years	*3 years*	*Males aged a years*	*Females aged*	
					a ~ 2 years	*a — 3 years*
(1)	(2)	(3)	(4)	(5)	CO	(7)
20	49.9	45.5	X	45.6	44.1	44.8
25	45.4	41.2	41.4	41.2	39.9	40.8
30	40.9	36.7	37.0	37.0	35.5	36.3
35	36.3	32.3	32.5	32.5	31.0	31.9
40	31.8	27.9	28.2	28.1	26,5	27.4
45	27.5	23.8	24.0	24.1	22.2	23.0
50	23.4	20.0	20.2	20.2	18.0	18.9
55	19.6	16.4	16.6	16.4	14.1	14.9
60	16.2	13.3	13.5	13.2	10.5	11.3
65	13.1	10.5	10.7	10.2	7.5	8.1
70	10.4	8.1	8.2	7.4	4.9	5.4
75	8.2	6.0	6.2	5.3	3.0	3.3

the remainder of his life is $e_m(a)$ and that can be achieved if the person remarries almost immediately following divorce or widowhood in the event of the dissolution of his marriage in one of those ways. Similarly, $A(a, b)$ has its

Upper limit no higher than the minimum of e_m *(a)* and e_f (b). Noting further that e_m *(a)* < e_f *(b)*, not only for the normative but also for all age differences shown in Table 2, the *A(a, b)* values in columns *3-4* of Table 3 may be compared with those of e_m *(a)*, which are given in column 2.

The number of years expected to be spent as married by males, i.e., *B(a, b)* are shown in column 5 of Table 3 for different ages. These values are conditional to a man's being married at the respective ages and their differences from those in column 2 provide the expected number of years in widowed and divorced states. In the last two columns of Table 3, similar values are shown for females who are two or three years younger than their husbands. These figures, compared with *A(a, b)*, show the net effect of divorce and remarriage on the expected number of years of marriage. Clearly, the magnitude of B(a, *b)* is reduced by divorce and is raised by remarriage. According to Table 3, these effects seem to cancel each other for the males, assuming of course that, on an average, the age difference between the spouses is about two or three years. However, the corresponding expected number of years for women are consistently smaller which leads to the suggestion that the balance between divorce and remarriage is less favorable for the females. In other words, compared with men, women either remarry less often or experience greater difficulty in remarriage or both.

7. Conclusion

In summary therefore, the average number of years lived as married by the U.S. males (including all marriages) would have remained pretty much the same if divorces and remarriages were both banned. The females, on the other hand, are spending fewer years of their lives as married, when the normative husband-wife age difference is held at two to three years. Thus, remarriage barely compensates for divorce among the males while its contribution to the total number of married years among the females is small compared to the loss suffered from divorce. This coupled with the facts that women are married to men who are older in age and that the life expectancy of women is greater than men (by about 7 years at birth for U.S. population), tends to lower the proportion of the life cycle that a woman lives as married.

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